Краткие сообщения ОИЯИ № 2[48]-91 *JINR Rapid Communications No.2[48]-91*
УДК 539.142

CLUSTERS IN THE NUCLEAR MODEL OF CHIRAL FIELD SOLITONS

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Within a variational approach to the Skyrme model the structure of the 4- and 6-baryon system is investigated. The calculated configurations of α-particle-like type appear to exist in an excited state in the structure of the 6-baryon system.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Кластеры в ядерной модели киральных солитонов

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В рамках вариационного приближения в модели Скирма исследована структура 4- и 6-барионных систем. В структуре 6-барионной системы обнаруживаются альфа-кластерные конфигурации, находящиеся в возбужденном состоянии.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

1. Introduction

With advances in accelerators and experimental techniques the study of nuclei lying far from β-stability continues to open the nuclear frontier, providing us with valuable insight into the properties of nuclei with extreme neutron to proton composition.

The ability to produce and separate or identify such exotic nuclei allows researches to use the nucleus as a microscopic laboratory in which interesting structural effects that occur in nuclei are investigated. For example, close to the drip line the occurrence of a pair of loosely bound neutrons may result in larger than expected matter radii. The results obtained at both BEVALAC and GANIL seem to support the neutronization of the nuclear surface in the form of a neutral halo around the nucleus that extends out to several times the nuclear radius. The experi-

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mental data on incident nucleus fragmentation reflect, to a great extent, the momentum distributions of nucleons and groups of nucleons in the incident nucleus and give a possibility of judging about the structure of the nucleus in general.

The theoretical approaches to the interpretation of these data can be catalogued into different categories: i) calculations, already existing applied to describe the experiments and ii) ad hoc models generated to explain the first data. It seems that the conventional approaches such as shell-model are not able to reproduce the data. These models underestimate the nuclear cross-sections as well as the nuclear radii deduced from measurements on light targets for most neutron-rich nuclei /1-3/.

The authors of /4-5/ carried out microscopic calculations of ^{6}He, ^{6}Li, ^{6}Be — nuclei in the frame of the $\alpha + 2N$ model with pairwise potentials fitted to phases of low-energy two-particle scattering. As a result two possible space configurations of the distributions of the nucleons have been obtained. The "dinucleon" type corresponds to the neutrons positioned close to one another on the side of the α-particle. The "cigar" type correspons to the neutrons located on the opposite sides of the α-particle. The single-nucleon matter r.m.s. radius turned out to be about 2.6 Fm which reproduces well the experimental data from /6/.

Receintly variational calculations of the structure of light nuclei have been carried out on the frame of the model of chiral solitons /7/. It has been shown that nuclei as chiral coliton states could have a complicated topological structure of the distribution of the baryon charge and mass density. Some of them could have isomer states which could differ by their form and size.

In this letter we present briefly the results of our variational calculations of the structure of classical solitons in the framework of the original SU(2)-Skyrme model /8/ for baryon number $B = 4{,}6$ and their r.m.s. radii. After the quantization procedure these states are to be identified with nuclei. We would like to emphasize that although the baryon charge distribution of baryon systems with $B \leq 6$ has been calculated in /9-10/, the essentially new point in our paper is the possibility of existence of isomers, in the form of light nuclei and their properties.

2. The Structure of Solitons in the Nuclear Model of Chiral Fields

In view of the fact that the minimum energy solutions up to $B = 6$ have been obtained by now without any limiting assumptions the variational approach can be useful when the constraints do not lead too far away from the true minimum configuration and add deeper insight to the nature of the true solution.

Recently a variational ansatz has been considered in [10-15]. The ansatz being very simple, gives the possibility of dealing analytically with the nuclear problem.

Here we follow our paper [15] with some modifications. We use more general assumption about the configuration of the isotopic vector field $\vec{N}$:

$$\vec{N} = \{\cos(\Phi(\phi, \theta)) \cdot \sin(T(\theta)),\ \sin(\Phi(\phi, \theta)) \cdot \sin(T(\theta)),\ \cos(T(\theta))\} \tag{1}$$

in variational form of the chiral field U:

$$U(\vec{r}) = \cos F(r) + i(\vec{\tau} \cdot \vec{N}) \sin F(r). \tag{2}$$

In eq.(1) $\Phi(\phi)$, $T(\theta)$ are some arbitrary functions of the angles (θ, ϕ) of the vector r in the spherical coordinate system. We consider the Lagrangian density $\mathcal{L}$ for the stationary solution:

$$\mathcal{L} = \frac{F_\pi^2}{16} \cdot Tr(L_k L_k) + \frac{1}{32e^2} \cdot Tr[L_k, L_k]^2 . \tag{3}$$

Here $L_k = U^+ \partial_k U$ are the left currents. After some tedious algebra, (1), (2), (3) lead to the expression

$$\mathcal{L} = \mathcal{L}_2 + \mathcal{L}_4, \tag{4}$$

where

$$\mathcal{L}_2 = -\frac{F_\pi^2}{8} \{ (\frac{\partial F}{\partial x})^2 + [(\frac{\sin T \partial \Phi}{\sin\theta\, \partial\phi})^2 + (\frac{\partial T}{\partial\theta})^2 + \sin^2 T (\frac{\partial\Phi}{\partial\theta})^2 \frac{\sin^2 F}{r^2} \tag{5}$$

and

$$\mathcal{L}_4 = -\frac{1}{2e^2} \cdot \frac{\sin^2 F}{r^2} \cdot \{ \frac{\sin^2 T}{\sin^2\theta} (\frac{\partial T}{\partial\theta})^2 (\frac{\partial\Phi}{\partial\phi})^2 \cdot \frac{\sin^2 F}{r^2} + \\ + [\frac{\sin^2 T}{\sin^2\theta} (\frac{\partial\Phi}{\partial\phi})^2 + (\frac{\partial T}{\partial\theta})^2 + \sin^2 T (\frac{\partial\Phi}{\partial\theta})^2] (\frac{\partial F}{\partial x})^2 \} . \tag{6}$$

Further on we follow our paper [17] in the derivation of the main equations of the model. Thus all solutions $U_{n\ell\{k(d)\}}$ are classified by a set of integer numbers n, ℓ and $k_0, \ldots, k_{\ell-1}$. The functions $F(x)$ and $T(\theta)$ have to obey the equations (14), (15) from [15] in arbitrary space region with given number k. Here we present the numerical results of our calculations of the structure of 4- and 6-baryon systems.

For that purpose we calculate the baryon charge density

$$J_0^B(\vec{r}) = -\frac{1}{24\pi^2}\cdot\epsilon_{0\mu\nu\rho}\,\mathrm{Tr}(L_\mu L_\nu L_\rho) \tag{7}$$

which is obtained to be

$$J_0^B(r,\theta) = -\frac{1}{2\pi^2}\cdot\frac{\sin^2 F}{r^2}\cdot\frac{dF}{dr}\cdot\frac{\sin T}{\sin\theta}\cdot\frac{dT}{d\theta}\cdot\frac{d\Phi}{d\phi}. \tag{8}$$

Equation (9) immediately results in the expression for the corresponding topological charge

$$B = n\cdot\sum_{m=0}^{\ell-1}(-1)^m k_m. \tag{9}$$

In Figs. 1, 2 we present some of the baryon charge distributions in the (X, Z) plane for solitons with different topological structure and baryon number B = 4 and 6. In [16] a series of toroidal solutions has been obtained. Such toroidal configurations are present in the structure of some of the heavier skyrmions. In fact the structure of the multiskyrmion objects with B = 4 and 6 is far more complicated.

The α-like object with B = 4 may consist of one toroidal soliton with B = 2 and two solitons with B = 1 (Fig.1a). This is the configuration of the soliton $U_{13\{1,1,2\}}$. Another possible configuration is $U_{12\{1,3\}}$ which consists of one toroid with B = 3 and a soliton with B = 1. The last three possible configurations are: $U_{11\{4\}}$ consisting of one configuration with B = 4, $U_{14\{1,1,1,1\}}$ consisting of two apple-like solitons and two toroidal solitons each with B = 1 (Fig. 1b), and $U_{12\{2,2\}}$ consisting of two toroidal solitons each with baryon charge 2 (Fig.1c).

In the structure of the 6-baryon system about 15 different configurations may appear. We will consider only the most interesting of them given in Table 2 and Fig.2.

As is seen from Tables 1, 2 the most compact configurations are these consisting of equal toroidal skyrmions. These are the configurations $U_{12\{2,2\}}$ for the 4-skyrmion system and $U_{12\{3,3\}}$ for

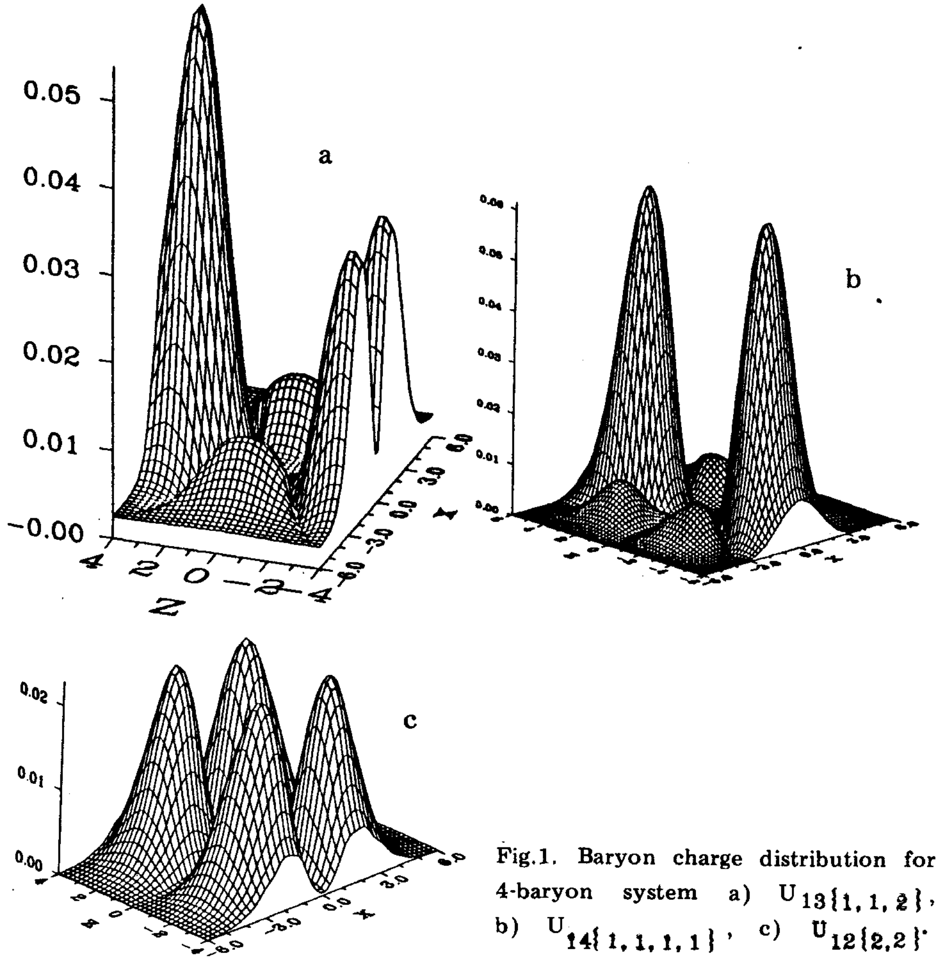

Fig.1. Baryon charge distribution for 4-baryon system a) $U_{13\{1,1,2\}}$, b) $U_{14\{1,1,1,1\}}$, c) $U_{12\{2,2\}}$.

Table 1. The calculated soliton masses in $(\pi F_\pi/e)$ units and r.m.s. radii in $1/F_\pi e$ units for baryon charge $B = 4$

$U_{n\ell\{k(d)\}}$	11{4}	12{1,3}	1,2{2,2}	13{1,1,2}	14{1,1,1,1}
M_U	47.67	46.53	45.53	54.52	71.16
$r.B^{1/2}$	8.26	8.54	7.68	8.50	8.19

the 6-skyrmion system. Their r.m.s. radii significantly differ from the radii of the other configurations, containing nontoroidal skyrmions.

It is remarkable that some of the toroidal solitons with $B = 1$ which take place in the construction of multiskyrmions have a deformed

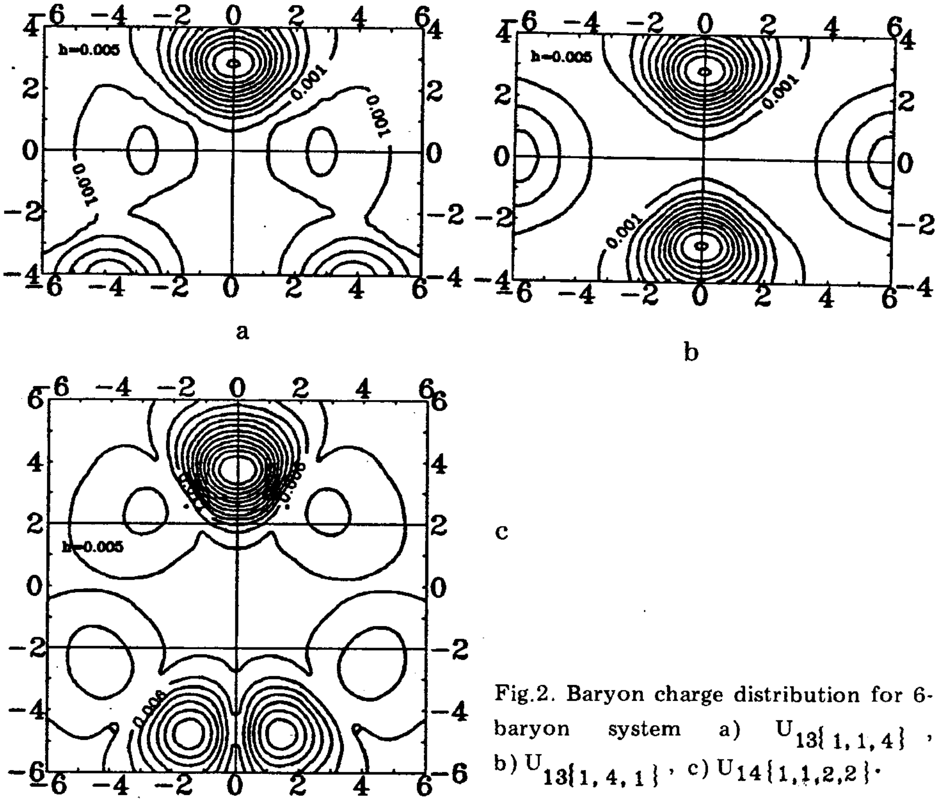

Fig.2. Baryon charge distribution for 6-baryon system a) $U_{13\{1,1,4\}}$, b) $U_{13\{1,4,1\}}$, c) $U_{14\{1,1,2,2\}}$.

Table 2. The calculated soliton masses in $(\pi F_\pi/e)$ units and r.m.s. radii in $1/F_\pi e$ units for baryon charge B = 6

$U_{n\ell\{k(d)\}}$	12{3,3}	12{2,4}	13{1,1,4}	13{1,4,1}	14{1,1,2,2}
M_U	66.69	67.41	72.41	82.30	88.88
$r.B^{1/2}$	11.3	12.00	13.7	13.67	12.45

structure. They exist only in the internal regions. Moreover the toroid with greater baryon charge B has greater radius (see Figs.).

One can find that two kinds of particle-like objects exist in the structure of the 6-skyrmion system. One of them is the compact α-particle consisting of two toroids each of baryon charge 2 (Fig.2c). The other kind is a swelling toroid with B = 4. The configurations $U_{13\{1,1,4\}}$ (a "dinucleon" type) and $U_{13\{1,4,1\}}$ (a "cigar" type) do not differ in size (Fig.2a,b). Their r.m.s. radius, calculated with values 109.45 and 4.138 for F and e, is far more close to the experimentally meaused one /6/ than the calculated r.m.s. radius for the configuration $U_{14\{1,1,2,2\}}$. So we can interpret this result as though the α-particle in He, Li exists in an exited state.

3. Summary

We summarize this letter by mentioning the following: within a variational approach to the Skyrme model the structure of the 4- and 6-baryon systems has been investigated. The calculated solitons have isomer states which differ by their form and size. Configurations of α-particle-like type appear to exist in the structure of the 6-baryon system. The calculated r.m.s. radii seem to correspond to the situation when the α-particle in the 6-baryon system exists in an exited state.

Acknowledgements

We thank Drs. M.K.Volkov, V.Yu.Novozhilov, D.I.Kazakov, R.Kalpakchieva for very useful discussions of our results.

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Received on April 9, 1991.